On slope unstable Fano varieties

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Slope stability

This is the joint work with Ching-Jui Lai.

Definition

Let A be a nef and big divisor on a normal projective variety X. For any torsion free sheaf $\mathcal{E}$ on X, we define the slope of $\mathcal{E}$ with respect to A as $\mu_A(\mathcal{E}) = \frac{c_1(\mathcal{E}) \cdot A^{n-1}}{\operatorname{rank}(\mathcal{E})}$. We say $\mathcal{E}$ is A-semistable (resp. A-stable) if $\mu_A(\mathcal{F}) \leq \mu_A(\mathcal{E})$ (resp. $\mu_A(\mathcal{F}) < \mu_A(\mathcal{E})$) for any nonzero proper subsheaf $\mathcal{F} \subset \mathcal{E}$.

We say $\mathcal{E}$ is slope unstable if it is not slope stable.

Motivation

Conjecture (Peternell)

The tangent bundle T_X of a Fano manifold X is $(-K_X)$-slope unstable only if the relative tangent bundle of a K_X-negative extremal contraction destabilizes it.

Known: $\dim X = 2$ by Fahlaoui and $\dim X = 3$ by Steffens.

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The tangent bundle of a Fano manifold X of Picard rank one is $(-K_X)$-slope stable.

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The tangent bundle of a Fano manifold X of Picard rank one is $(-K_X)$-slope stable.

It holds in dim ≤ 5 by Peternell-Wiśniewski and Hwang.
Kanemitsu gives a counterexample when $\dim X = 14$.

$$\begin{array}{ccc} \mathrm{Bl}_Z X & \xrightarrow{\pi} & X \\ {\scriptstyle f}\downarrow & & \\ Y & & \end{array}$$

Maximal destabilizing sheaf

Let X be a weak $\mathbb{Q}$-Fano variety.
Assume T_X is not $(-K_X)$-slope stable.
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Theorem (Campana-Păun, Ou)

$\mathcal{F}$ is an algebraically integrable foliation with rationally connected leaves.

Foliation

Definition (Foliation)

Let X be a normal variety. A foliation $\mathcal{F}$ of rank r on X is a rank r coherent subsheaf of the tangent sheaf $\mathcal{T}_X$ such that

1. $\mathcal{F}$ is closed under the Lie bracket and
2. $\mathcal{F}$ is saturated, that is $\mathcal{T}_X/\mathcal{F}$ is torsion-free.

Remark

1. By Frobenius' theorem, there are leaves of $\mathcal{F}$.
2. $\mathcal{F}$ is uniquely determined on a dense open subset of X.

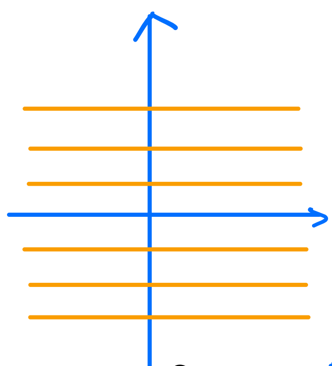
The canonical divisor $K_{\mathcal{F}}$ is defined as $\mathcal{O}(K_{\mathcal{F}}) = \det(\mathcal{F}^*)$.

Examples: Foliations

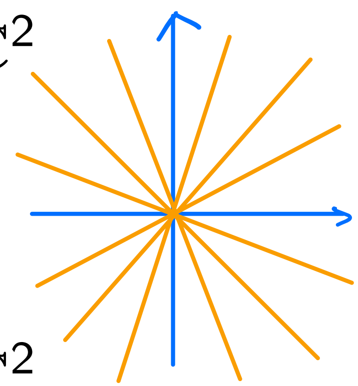
1. $f : X \dashrightarrow Y$ a dominant rational map.

Algebraically integrable

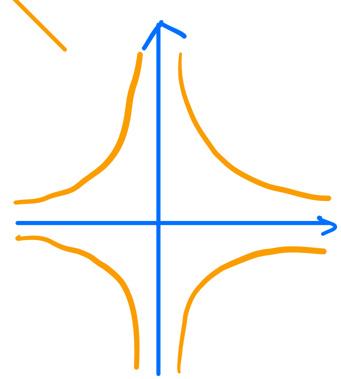
2. $v_1 = \frac{\partial}{\partial x}$ on $\mathbb{C}^2$



3. $v_2 = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}$ on $\mathbb{C}^2$



4. $v_3 = x\frac{\partial}{\partial x} - y\frac{\partial}{\partial y}$ on $\mathbb{C}^2$



Invariance

Let X be a normal variety and $\mathcal{F}$ be a foliation on X.
We say a subvariety $W \subset X$ is invariant under $\mathcal{F}$ if $\mathcal{F}|_W \to T_X|_W$ factors through T_W.

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"W is the union of some leaves"

Foliation singularities

Let $f\colon Y \to X$ be a proper birational morphism and $\mathcal{F}$ be a foliation on X. Suppose $K_{\mathcal{F}}$ is $\mathbb{Q}$-Cartier. For any $E \subset Y$ exceptional over X, we define the foliated discrepancy

$$a(E, \mathcal{F}) := \mathrm{ord}_E(K_{f^*\mathcal{F}} - f^*K_{\mathcal{F}}).$$

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We say

$$x \text{ is } \begin{cases} \text{terminal} & \text{if } a(E,\mathcal{F}) > 0 \\ \text{canonical} & \text{if } a(E,\mathcal{F}) \geq 0 \\ \text{log terminal} & \text{if } a(E,\mathcal{F}) > -\varepsilon(E) \\ \text{log canonical} & \text{if } a(E,\mathcal{F}) \geq -\varepsilon(E) \end{cases} \quad \text{for every divisor } E \text{ over } x$$

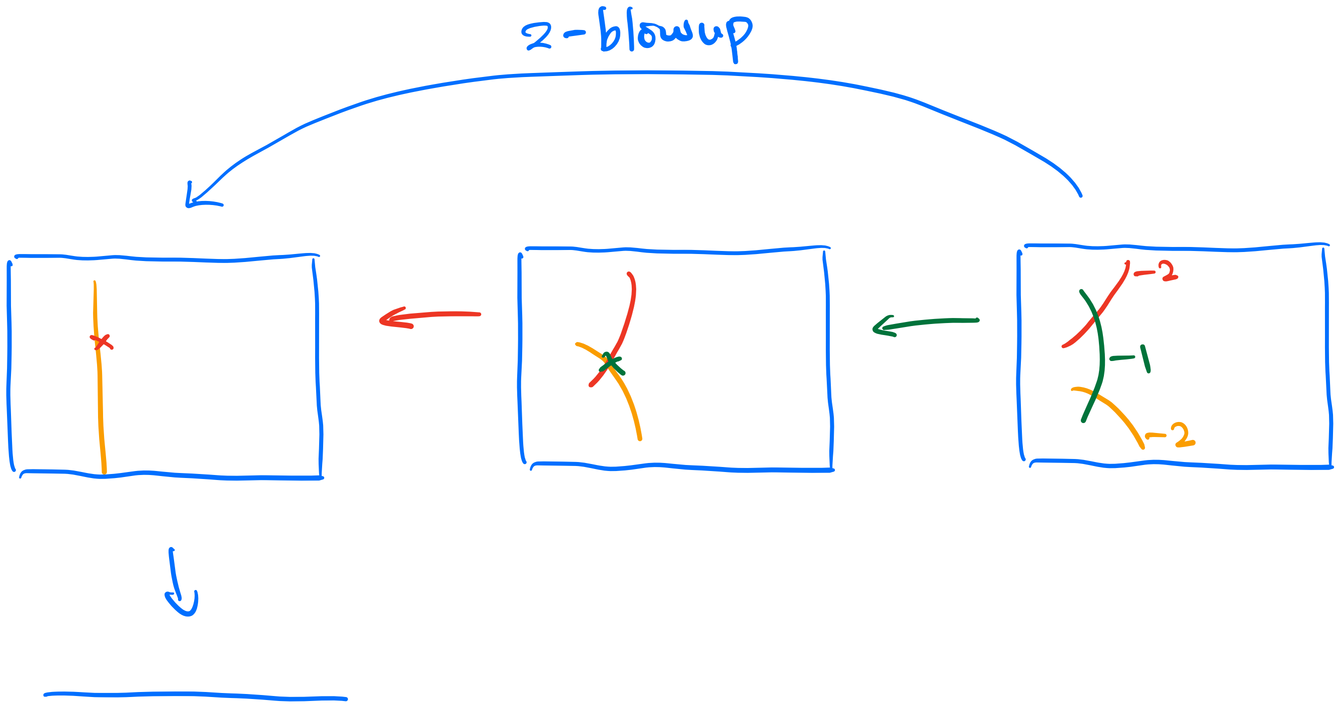
$(K_{\mathcal{F}} + \varepsilon(E)E)|_E = K_{\mathcal{F}_E} + \mathrm{Diff}.$

where $\varepsilon(E) = 0$ if E is $f^*\mathcal{F}$-invariant and $\varepsilon(E) = 1$ if E is not $f^*\mathcal{F}$-invariant.

Main Theorem

Theorem (C.-Lai)

Let X be a smooth weak del Pezzo surface, that is $-K_X$ is nef and big. If T_X is $(-K_X)$-slope unstable, then X is obtained from the Hirzebruch surface $\mathbb{F}_n$ with $n \in \{0, 1, 2\}$ by at most three "2-blowups" at general points. Moreover, the maximal destabilizing sheaf is the foliation induced by its canonical morphism to $\mathbb{P}^1$.



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Corollary

Let X be a weak del Pezzo surface with at worst canonical singularities and whose tangent sheaf T_X is $(-K_X)$-slope unstable. If $\mathcal{F}$ is the maximal destabilizing sheaf, then $(X, \mathcal{F})$ is induced from its minimal resolution, which is one of the surfaces above.

$\pi : \tilde{X} \to X$ $\qquad$ $K_X^2 = K_{\tilde{X}}^2$

$\pi^* K_X = K_{\tilde{X}}$ $\qquad$ $K_{\mathcal{F}} \cdot K_X = K_{\tilde{\mathcal{F}}} \cdot K_{\tilde{X}}$

Idea of Proof

Proposition

Let X be a weak $\mathbb{Q}$-Fano variety whose tangent sheaf is $(-K_X)$-unstable. Suppose $\mathcal{F}$ is the maximal destabilizing sheaf. Then $K_{\mathcal{F}}$ is NOT pseudo-effective.

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Suppose not, then $\mu(\mathcal{F}) = \frac{c_1(\mathcal{F})\cdot(-K_X)^{n-1}}{\operatorname{rank}\mathcal{F}} = \frac{(-K_{\mathcal{F}})\cdot(-K_X)^{n-1}}{\operatorname{rank}\mathcal{F}} \leq 0$, contradicting to $\mu(\mathcal{F}) \geq \mu(T_X) > 0$.

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1. Run a $K_{\mathcal{F}}$-MMP. So we have $X \to X_+$ where $X_+ \to Z_+$ is a Mori fiber space.

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1. Run a $K_{\mathcal{F}}$-MMP. So we have $X \to X_+$ where $X_+ \to Z_+$ is a Mori fiber space.
2. $\dim Z_+ = 1$: Classification.
3. $\dim Z_+ = 0$: Non-existence.

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Let X be a normal $\mathbb{Q}$-factorial surface and $\mathcal{F}$ be a foliation on X. If $\varphi\colon X \to Y$ is a morphism contracting a $K_{\mathcal{F}}$-negative extremal curve C, then C is $\mathcal{F}$-invariant.

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Proof

Suppose not.

$$0 > (K_{\mathcal{F}} + C) \cdot C = \deg(K_{\mathcal{F}_C} + \operatorname{Diff}(\mathcal{F}, 0)) = \deg \operatorname{Diff}(\mathcal{F}, 0) \geq 0,$$

which is impossible.

$\dim Z_+ = 1$: Classification

$$\begin{array}{ccc} \varphi_+ : X & \longrightarrow & Y_+ \\ & & \Big\downarrow {\scriptstyle f_+} \; {\scriptstyle K_{\mathcal{F}}\text{-Mfs}} \\ & & Z_+ \end{array}$$

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$$\varphi_+ : X \xrightarrow{\varphi} \widetilde{Y_+} \xrightarrow{\varphi'} Y_+ \qquad Y_+ \xrightarrow[f_+]{K_{\mathcal{F}}\text{-Mfs}} \mathbb{P}^1$$

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2. Now we assume $m > 0$. Let $N_i = 2K_{\mathcal{F}_i} \cdot K_{X_i} - K^2_{X_i}$ for $0 \leq i \leq m$. Note that $N_0 = 0$ and $N_m \geq 0$ by assumption.

$= 2(\mu(\mathcal{F}_i) - \mu(T_{X_i}))$

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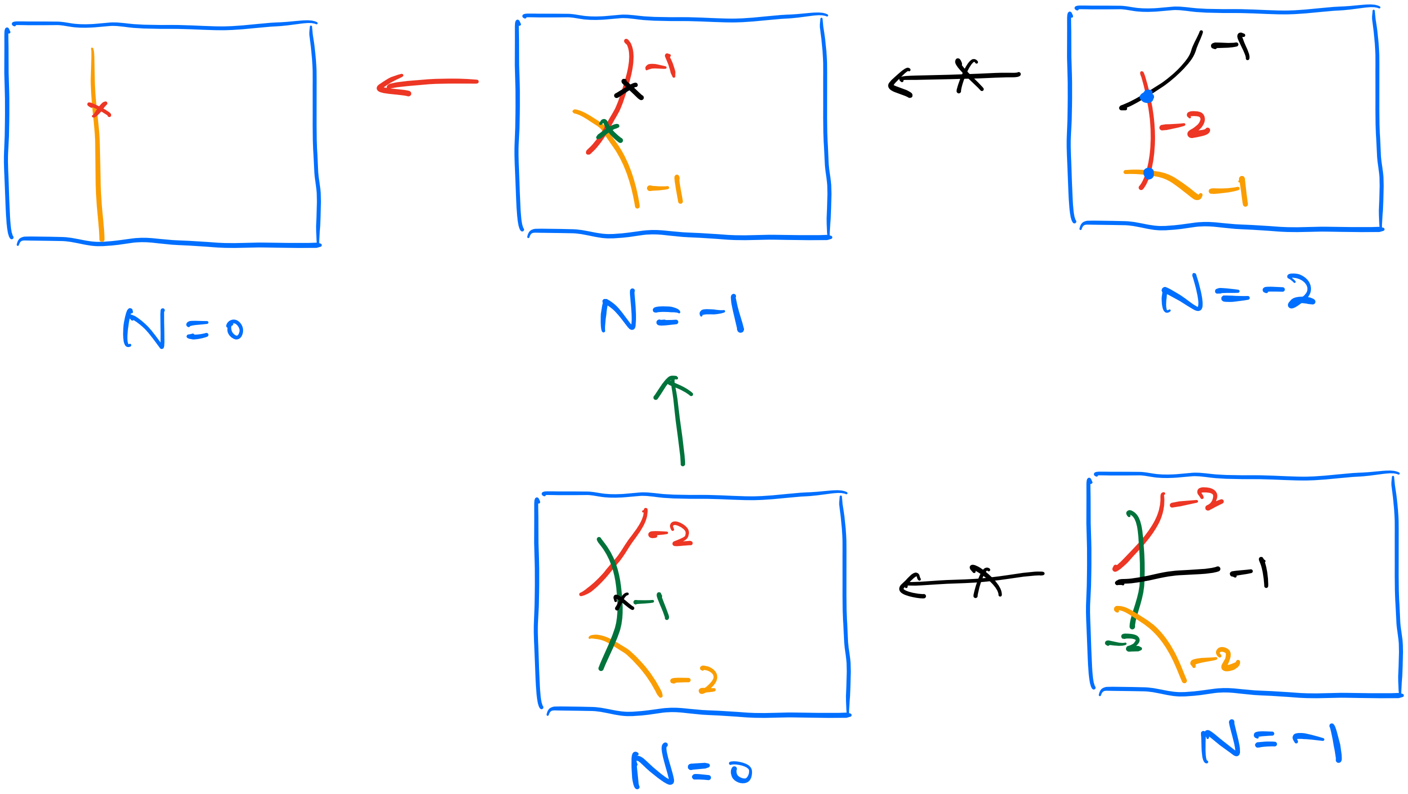
Lemma

$$N_{i+1} = \begin{cases} N_i - 1 & \textit{if } x \notin \operatorname{Sing} \mathcal{F} \\ N_i + 1 & \textit{if } x \in \operatorname{Sing} \mathcal{F} \end{cases}$$

dim $Z_+ = 1$: Classification (Cont'd)



Each 2-blowup decrease K_X^2 by 2.

As $K_{\mathbb{F}_n}^2 = 8$, we can have at most 3 2-blowups.

$\dim Z_+ = 0$: Non-existence

$X \to Y_+$ $\rho(Y_+) = 1$ [AD]

$-K_{Z_+}$ ample

Theorem (Ambro-Cascini-Shokurov-Spicer)

Let X be a normal variety and $\mathcal{F}$ be an algebraically integrable foliation. Then there exists a modification of $\mathcal{F}$, that is a birational morphism $\pi\colon W \to X$ such that

$W \xrightarrow{\pi} X$, $g\colon W \to Z$

1. *W is $\mathbb{Q}$-factorial and klt,*
2. *$\mathcal{G} := \pi^{-1}\mathcal{F}$ is induced by an equi-dimensional morphism $g\colon W \to Z$ over a smooth variety Z.*
3. *$(\mathcal{G}, \sum \varepsilon(E)E)$ is log canonical where the sum is over all π-exceptional prime divisors.*
4. *$K_{\mathcal{G}} + \sum \varepsilon(E)E + G = \pi^* K_{\mathcal{F}}$ for some effective and π-exceptional divisor G.*

dim $Z_+ = 0$: Non-existence (Cont'd)

Theorem (C.-Lai)

$\mathcal{F}_+$ is a Fano toric foliation on a toric variety Y_+ with at worst canonical singularities, of $\rho(Y_+) = 1$, and $|\mathrm{Sing}(Y_+)| \leq 2$.

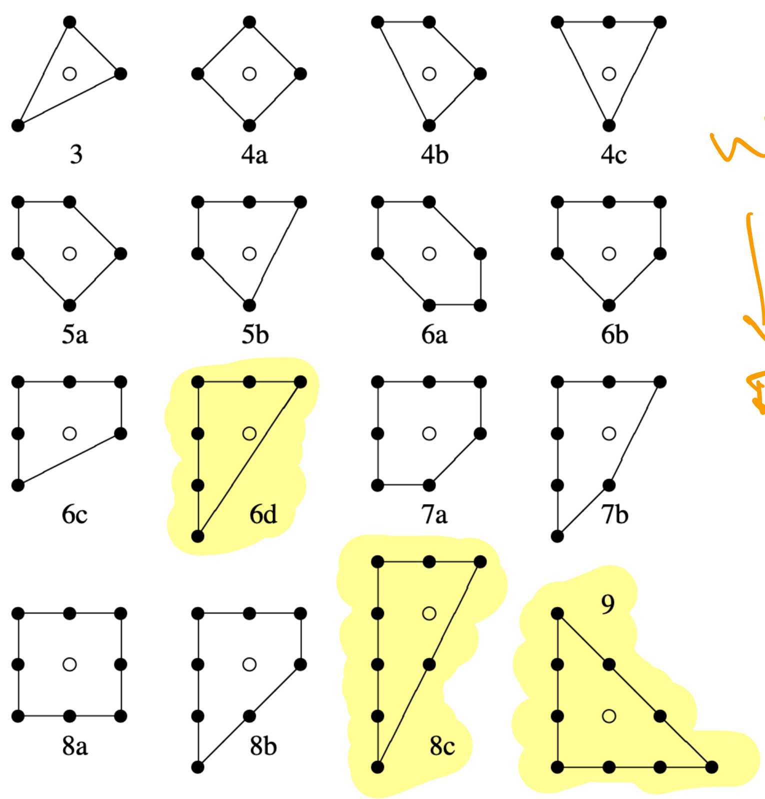
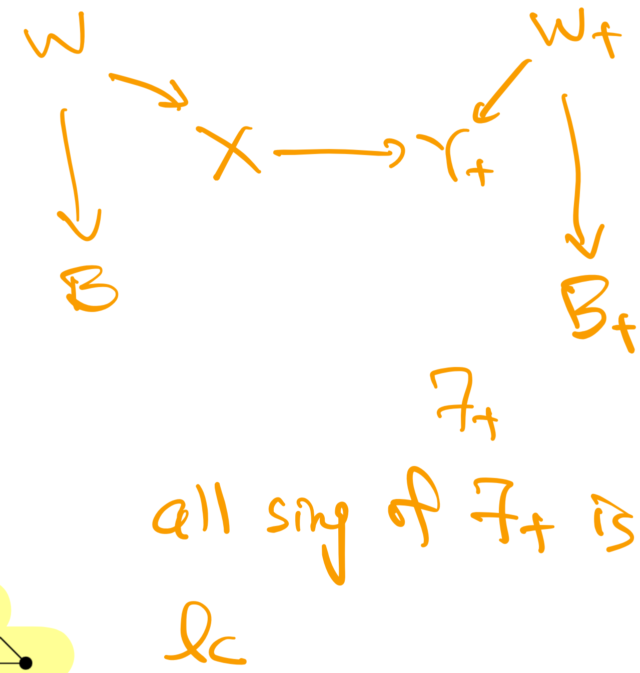


Figure 2. The 16 equivalence classes of reflexive lattice polygons in $\mathbb{R}^2$



$\dim Z_+ = 0$: Non-existence (Case 9)

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$(-1)\,(-3)\ \ (-3)^2$

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Lemma

There is no φ_+-exceptional divisor over p.

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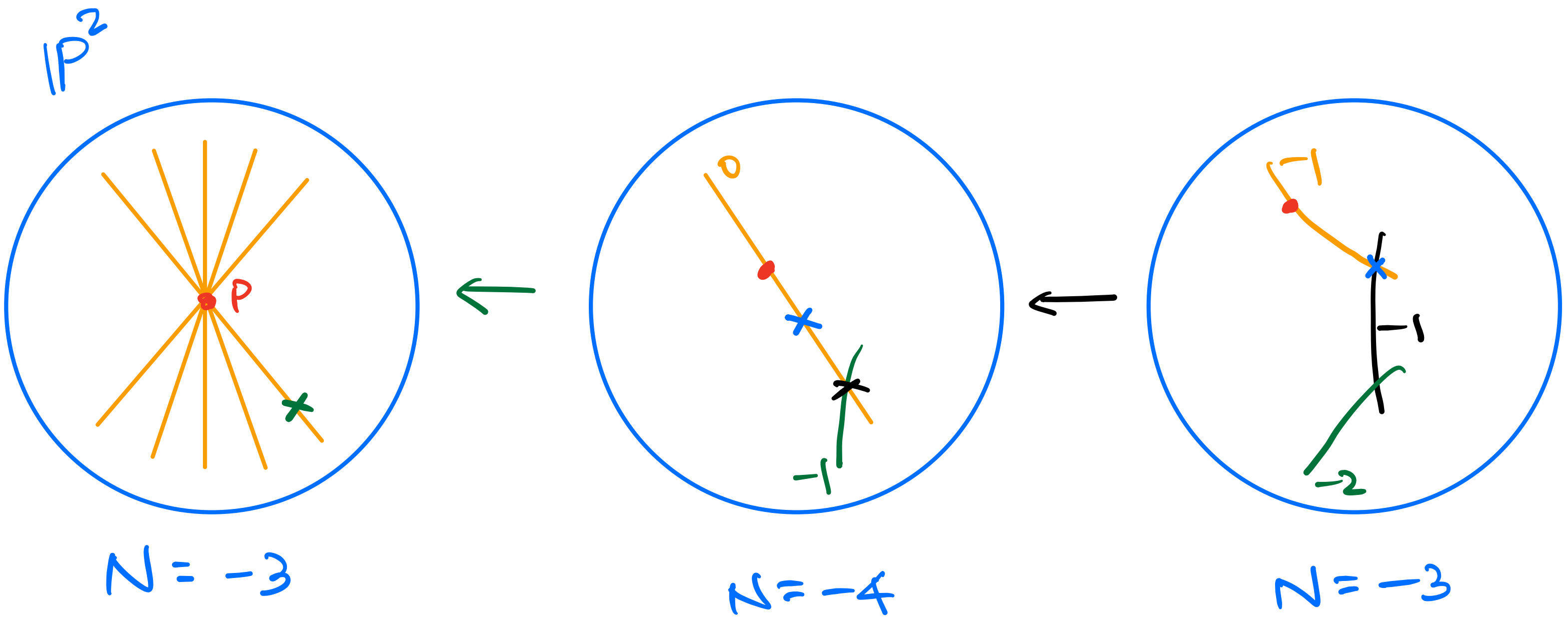
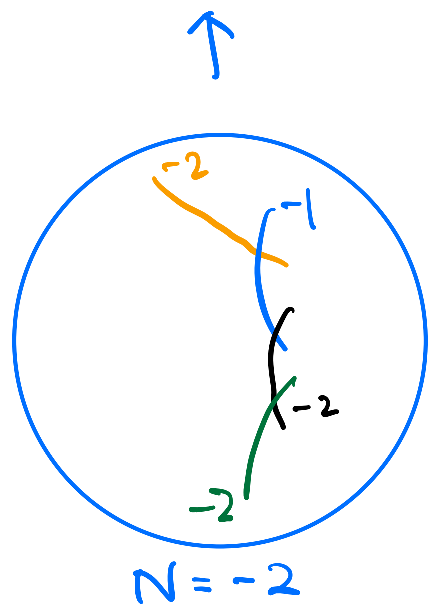
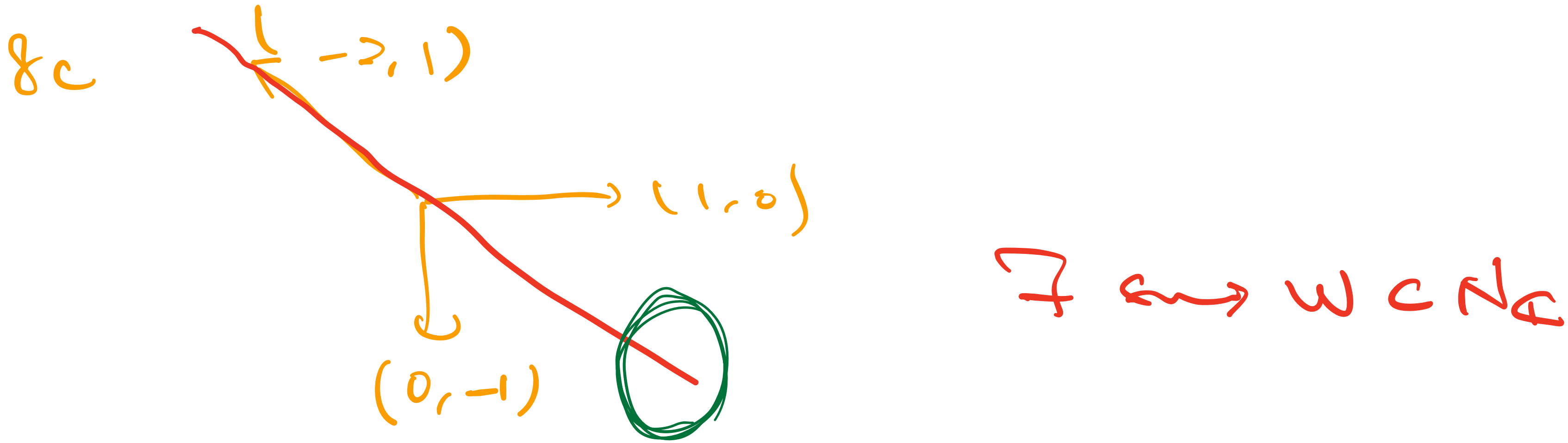




3 blowups increase N by 1.

We need 9 blowups to achieve N=0.

However. $K_X^2 = K_{\mathbb{P}^2}^2 - 9 = 0$



Thank you.